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A generalization of Jensen's inequality for polynomials having concentration at low degrees

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ABSTRACT

In this paper we give some asymptotic estimates for the best lower bound $C(d, k, p)$ of the Jensen's functional $J(f) = \int_0^{2\pi} \log|f(e^{i\theta})| \frac{d\theta}{2\pi}$ for when the polynomial f has a concentration d at low degrees k measured by the l_p-norm under the L_p-norm, $p > 2$.

1. Introduction

Let $f(z) = \sum_{j=0}^{n} a_j z^j \neq 0$ be a polynomial with complex coefficients and with the L_p-norm $\|f\|_{L_p} = \left(\int_0^{2\pi} |f(e^{i\theta})|^p \frac{d\theta}{2\pi}\right)^{\frac{1}{p}}$. Next, let d be a real number such that $0 < d < 1$. We say that $f(z)$ has a concentration d of degree at most k, measured by the l_p-norm under the L_p-norm, $p > 2$, if the following inequality holds:

$$\left(\sum_{j\leqslant k} |a_j|^p\right)^{\frac{1}{p}} \geqslant d \cdot \left(\int_0^{2\pi} \left|f\left(e^{i\theta}\right)\right|^p \frac{d\theta}{2\pi}\right)^{\frac{1}{p}} = d \cdot \|f\|_{L_p}. \tag{1}$$

The condition (1) can be imposed because

$$\|f\|_{L_p} > |f|_{l_p} \geqslant \left(\sum_{j\leqslant k} |a_j|^p\right)^{\frac{1}{p}}$$

whenever $p > 2$, where $|f|_{l_p} = \left(\sum_{j=0} |a_j|^p\right)^{\frac{1}{p}}$.

Other ways of measuring such a concentration can be expressed. For instance,

$$\left(\sum_{j\leqslant k} |a_j|^p\right)^{\frac{1}{p}} \geqslant d \cdot \left(\sum_{j\geqslant 0} |a_j|^p\right)^{\frac{1}{p}}, \quad p \geqslant 1 \tag{2}$$

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